

TABLE II. Corrections to the one-phonon "hopping" probability and lattice relaxation energy for *n*-type Ge and Si.

Material	$\alpha_{T \rightarrow 0}$	$A_{T \rightarrow 0}$	$(W_{ba}^{tot}/W_{ba}^1)_{T \rightarrow 0}$	E_{LR} (eV)
<i>n</i> -Ge	0.059(0.08)	0.067(0.08)	0.82(0.78)	$2.5 \times 10^{-5}(3.3 \times 10^{-5})$
<i>n</i> -Si	0.43(0.56)	0.39(0.59)	0.24(0.10)	$0.80 \times 10^{-3}(1.1 \times 10^{-3})$

difference of the electrostatic energy of the carrier in the two donors, v the velocity of sound in the crystal, and d the density of the crystal.

In the Table II we give the values of $\alpha_{T \rightarrow 0}$, $A_{T \rightarrow 0}$ and $(W_{ba}^{tot}/W_{ba}^1)_{T \rightarrow 0}$ obtained from (28)–(30) and from data given in Table I for *n*-type Ge and Si samples studied in Ref. 2. We give also the values of relaxation

energy E_{LR} of the lattice deformed by interaction with the donor electron, which is²

$$E_{LR} = \frac{E_1^2}{4\pi v^2 d} \int_0^\infty U(\tau) \tau^2 d\tau. \quad (31)$$

In Table II we give in brackets for comparison the values obtained previously² in approximation of hydrogen-like, spherical donor.

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Measurement of the Conductivity Effective Mass in Semiconductors Using Infrared Reflection

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There exist various approximate relationships between the conductivity effective mass of free carriers m_c and the angular frequency ω_0 of the reflectivity minimum in the infrared arising from the free-carrier dispersion. A detailed analysis of the reflectivity equation shows that it is possible to obtain a relationship between m_c and ω_0 when in addition one has a knowledge of the free-carrier concentration, the drift mobility, and the dielectric constant of the semiconductor at very high frequencies. This relationship is shown to yield a value of $m_c^* = 0.145$ for *n*-type germanium at room temperature using data presented in the literature. This value compares well with the value of $m_c^* = 0.15$ obtained using a combination of reflection and transmission measurements. The experimental work required to obtain accurate values for m_c using this relationship is considerably less than that required by other techniques, such as cyclotron resonance, magnetoplasma, and combined infrared reflection and transmission measurements, which are presently used. Moreover, this procedure can be applied over very broad ranges of temperature and of free-carrier concentration.

INTRODUCTION

IN this paper¹ we are concerned with the determination of the conductivity effective mass, which is understood to be the effective mass entering in the free-carrier mobility

$$\mu = (e/m_c)\langle\tau\rangle, \quad (1)$$

where e is the electronic charge, m_c is the conductivity effective mass, and $\langle\tau\rangle$ is the relaxation time averaged over all of the free current carriers. The determination of this parameter has received increasing attention in recent years. The earliest and the most extensive investigations to date have utilized cyclotron resonance

techniques. For example, the conductivity effective mass has been measured at liquid-helium temperature, where the relaxation time of free carriers is sufficiently long for both germanium and silicon.^{2,3} Recently infrared techniques also have been employed for this determination. In one instance, the conductivity effective masses in germanium, silicon, and indium antimonide were obtained from a combination of reflection and transmission measurements conducted at room temperature.⁴ In another instance only reflection measurements were required, in conjunction with varying intensities of magnetic field, to determine the conductivity effective masses of mercury selenide and indium antimonide.⁵

We are proposing in this paper that it is possible to determine m_c very accurately simply from a measure-

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² G. Dresselhaus, A. Kip, and C. Kittel, *Phys. Rev.* **98**, 368 (1955).

³ R. Dexter, H. Zeiger, and B. Lax, *Phys. Rev.* **104**, 637 (1956).

⁴ W. Spitzer and H. Fan, *Phys. Rev.* **106**, 882 (1957).

⁵ B. Lax and G. Wright, *Phys. Rev. Letters* **4**, 16 (1960).

ment of the frequency of the reflectivity minimum in the far infrared. Through a straightforward differentiation of the reflectivity equation, utilizing the dispersion relations, and the evaluation of this derivative at the minimum, a relationship is obtained which can be solved for m_c using an iterative procedure.

FREE-CARRIER DISPERSION

In the far infrared, at wavelengths longer than that of the intrinsic absorption edge, electromagnetic radiation interacts with the free carriers of the material and undergoes dispersion. The theory of this free-carrier dispersion is developed in detail in a recent book by Moss.⁶ He has shown that both classical and quantum mechanical analyses lead to the following relations for the real refractive index n and the absorption index (extinction coefficient) k of free carriers of one type only (i.e., holes or electrons):

$$n^2 - k^2 - \epsilon_\infty = \frac{Ne^2}{m\epsilon_0} \frac{\tau^2}{1 + \omega^2\tau^2} \quad (2)$$

and

$$2nk\omega = \frac{Ne^2}{m\epsilon_0} \frac{\tau}{1 + \omega^2\tau^2}, \quad (3)$$

where ϵ_∞ is the relative dielectric constant of the medium in the absence of any contribution from free carriers at very high frequencies, N is the free-carrier concentration, ϵ_0 is the permittivity of free space, m is the effective mass of the free carriers, and ω is the angular frequency of the infrared radiation.

Several points should be noted with regard to Eqs. (2) and (3). It has been shown for the energy-band model in which ellipsoidal surfaces of constant energy lie along rotation axes of the cubic point group, that the effective mass which enters in these expressions is the conductivity effective mass, m_c .⁷ In this case, m_c is defined by the relation $1/m_c = \frac{1}{3}(1/m_l + 2/m_t)$, where m_l is the value of the effective-mass tensor associated with the longitudinal axis of the ellipsoidal energy surface and m_t is that associated with each of the two transverse axes of the surface. Secondly, we note that the relaxation time τ is assumed to be independent of frequency and that the total frequency dependence is exhibited explicitly by the quantity ω . This assumption is not restrictive since the relaxation time is determined by the temperature of the crystal, its energy-band structure, and the density of imperfections in the lattice. Finally, it should be noted that the relaxation time τ is necessarily assumed to be energy-independent. This follows from the fact that the terms containing τ have not been averaged, $\langle \rangle$, with respect to the distribution function. This last assumption requires additional con-

sideration. For example, we wish to establish the conditions under which τ may be replaced by $\langle \tau \rangle$ in Eqs. (2) and (3). The details of the analysis for establishing these conditions are presented in Appendix A. However, in summary if the relaxation time is assumed to be proportional to a power of the free-carrier kinetic energy $\tau = \tau_0 E^s$, it is possible to compare the exact terms $\langle \tau^2 / (1 + \omega^2 \tau^2) \rangle$ and $\langle \tau / (1 + \omega^2 \tau^2) \rangle$ with the more convenient substitute terms $\langle \tau^2 \rangle / (1 + \omega^2 \langle \tau^2 \rangle)$ and $\langle \tau \rangle / (1 + \omega^2 \langle \tau^2 \rangle)$. Such a comparison shows that the latter terms can be substituted for the former under the following conditions with no introduction of error:

(i) when τ is independent of energy, $s=0$, as in the case of neutral impurity scattering,

(ii) when there is total degeneracy and the kinetic energy of all carriers may be considered to be the Fermi energy E_f .

The substitution can be effected under the following conditions with an introduction of error $< 13\%$:

(iii) when $s = -\frac{1}{2}$, as in the case of scattering arising from acoustical modes and from dislocations, for all degrees of degeneracy where $\omega\tau \geq 1$ while the degree of degeneracy required increases from $\eta \leq -4$ to $\eta = 0$ as $\omega\tau$ decreases below unity,

(iv) when $s = \frac{1}{2}$, as in the case of optical-mode scattering above the Debye temperature, for all degrees of degeneracy and for all values of $\omega\tau$, and

(v) when $s = \frac{3}{2}$, as in the case of scattering by ionized impurities, only under the condition of total degeneracy. These conditions are not considered to be overly restrictive but, in fact, to be rather generally satisfied. This consideration follows particularly in view of the fact that in order to observe the free-carrier reflectivity minimum at convenient wavelengths ($\approx 30 \mu$), it is often necessary to utilize free-carrier concentrations which result in at least partial, if not total, degeneracy. For these reasons, in our further consideration of Eqs. (2) and (3), we will assume that one of the conditions (i)–(v) is fulfilled and that τ may be replaced by $\langle \tau \rangle$ with an introduction of error $< 13\%$.

At this point, it must be recognized that the total dispersion of incident radiation in the far infrared arises from interactions with the free carriers and with the lattice as well. This latter dispersion is important in polar crystals. The complete dispersion relations, which include the contributions of both the free carriers and the lattice of a polar crystal AB , may be expressed as follows^{8,9}:

$$n^2 - k^2 = \epsilon_\infty - \frac{Ne^2}{m_c \epsilon_0 \omega^2} \frac{\omega^2 \langle \tau \rangle^2}{(1 + \omega^2 \langle \tau \rangle^2)} + \left[\frac{\epsilon_\infty + 2}{3} \right]^2 \frac{N_l Q^2}{M \epsilon_0} \left[\frac{(\omega_l^2 - \omega^2) \tau_l^2}{\omega^2 + (\omega_l^2 - \omega^2) \tau_l^2} \right] \quad (4)$$

⁶ T. Moss, *Optical Properties of Semiconductors—a Semiconductor Monograph* (Butterworths Scientific Publications, London, 1959), Chap. 2, p. 29.

⁷ Reference 6, p. 31.

⁸ Reference 6, Chaps. 2 and 16.

⁹ B. Szigeti, *Trans. Faraday Soc.* **45**, 155 (1949).

and

$$2nk = \frac{Ne^2}{m_c \epsilon_0 \omega^2} \frac{\omega \langle \tau \rangle}{(1 + \omega^2 \langle \tau \rangle^2)} + \left[\frac{\epsilon_\infty + 2}{3} \right]^2 \frac{N_i Q^2}{M \epsilon_0} \left[\frac{\omega \tau_l}{\omega^2 + (\omega_l^2 - \omega^2) \tau_l^2} \right]. \quad (5)$$

In both Eqs. (4) and (5) the last term is the contribution made by the ions of the crystal. The parameters in the other terms have been defined following Eqs. (2) and (3). The new parameters are as follows: N_i is the concentration of atom pairs AB ; Q is an effective charge defined by $Q = SZe$, where e is the electronic charge, Z is the valence of the compound, and S is a measure of the extent to which the ions of charge Ze are capable of being polarized; M is the reduced atomic mass $1/M = 1/M_A + 1/M_B$; ω_l is the angular frequency of the transverse lattice vibrations (Reststrahlen); and τ_l is a relaxation time related to the damping of the lattice vibrations.

The relative contribution of the lattice to the total dispersion varies from one semiconductor to another. Moreover, it even varies for a single semiconductor depending upon the free-carrier concentration. Consequently, this contribution must be evaluated for each individual case in order to determine if its effect is significant in the over-all dispersion. It is clear, however, that in several instances the lattice polarization may be neglected when analyzing the dispersion in the far infrared. For example, lattice polarization does not enter in connection with the elemental semiconductors, germanium and silicon. Moreover, a combination of several conditions, such as, a small value for ϵ_∞ , a large reduced mass M , a slight polarizability ($S \ll 1$), and a large free-carrier concentration ($N \gtrsim 10^{18}$ carriers/cm³), can serve to make the effect of the lattice polarization completely negligible.

Because various combinations of these conditions are quite often satisfied, the total dispersion in the far infrared can oftentimes be considered to be described by the simpler Eqs. (2) and (3). In the subsequent analysis we will make this assumption, namely, that the lattice contribution to the total dispersion is negligible in the frequency range of the reflectivity minimum arising from free-carrier absorption.

REFLECTIVITY MINIMUM

The reflectivity of a material for normally incident radiation upon the surface of an infinitely thick sample is given by¹⁰

$$R = \frac{[(n-1)^2 + k^2]}{[(n+1)^2 + k^2]}, \quad (6)$$

where n and k are described by Eqs. (2) and (3). This equation has been differentiated with respect to the variable Ω after rewriting the dispersion relations in the

form:

$$n^2 - k^2 = \epsilon_\infty - B/(1 + \Omega^2) \quad (7)$$

and

$$nk = B/2\Omega(1 + \Omega^2), \quad (8)$$

where

$$B = Ne^2 \langle \tau \rangle^2 / m_c \epsilon_0 \quad \text{and} \quad \Omega = \omega \langle \tau \rangle$$

and assuming that $\langle \tau \rangle$ is not a function of frequency. The condition of zero slope at the reflectivity minimum then yields an equation for the effective mass in terms of the frequency of the reflectivity minimum and other measurable parameters. The equation that results is the following:

$$m_c^*{}^3 - \frac{3\epsilon_\infty - 1}{4\epsilon_\infty(\epsilon_\infty - 1)} \frac{(1/\Omega^2) + 5 + 8\Omega^2}{(1 + 3\Omega^2)} C m_c^*{}^2 + \frac{3\epsilon_\infty - 2}{2\epsilon_\infty(\epsilon_\infty - 1)^2} \times \frac{(1 + 2\Omega^2)}{(1 + 3\Omega^2)} C^2 m_c^* + \frac{C^3}{4\epsilon_\infty(\epsilon_\infty - 1)^2(1 + 3\Omega^2)} = 0, \quad (9)$$

where $m_c^* = m_c/m_0$, m_0 is the free-electron mass, $C = Ne^2/m_0 \epsilon_0 \omega_0^2$, $\Omega = \omega_0 \langle \tau \rangle$, and ω_0 is the angular frequency of the reflectivity minimum. [The derivation of this equation is outlined in Appendix B. It is based entirely on Eqs. (6)–(8), and therefore depends on their applicability to a particular material system for its own validity.]

This equation is quite general in that no approximations are made with respect to the magnitude of Ω nor with respect to the magnitude of ϵ_∞ . [However, it is true that Ω must satisfy the conditions under which the substitution of $\langle \tau \rangle$ into Eqs. (2) and (3) was effected.]

We conclude that there are very broad conditions for which Eq. (9) represents an exact relationship, or very nearly so, between the conductivity effective mass and the frequency of minimum reflectivity and other measurable parameters. Equation (9) can therefore be used for the determination of m_c . This can be illustrated by considering the reflection curve for n -type germanium reported by Spitzer and Fan (S-F) and which is reproduced here as Fig. 1. The sample investigated by S-F had a carrier concentration of 3.9×10^{18} electrons/cm³, $\mu_h = 520$ cm²/V-sec,¹¹ $\epsilon_\infty = 16$ and a reflectivity minimum at $\lambda_0 \approx 23 \mu$. By making an initial estimate for the conductivity effective mass, say $m_c^* = 0.12$ as determined by cyclotron resonance, one can obtain an initial estimate for Ω . Using this value of Ω and the values for N , ϵ_∞ , and ω_0 listed above, Eq. (9) can be solved for m_c^* . This enables one to calculate a revised estimate for Ω and the procedure is repeated until the solution of Eq. (9) yields the same value for m_c^* as

¹¹ It is recognized that we are here employing a Hall mobility when actually the drift mobility is called for. However, according to the S-F report, the Hall coefficient was measured at 77°K where the Fermi level is $\approx 4.5 kT$ above the conduction band edge for this free-carrier concentration. It is reasonable to conclude that the mobility deduced from $R\sigma$ is very nearly the drift mobility.

¹⁰ Reference 6, p. 6.

before. In actual fact, because Ω^2 is large with respect to unity ($\Omega^2=13$) and therefore Eq. (9) is almost independent of Ω , this procedure converges very quickly. The value obtained for the conductivity effective mass using this procedure for the above example is $m_c^*=0.145$. This compares very well with the value $m_c^*=0.15$ calculated by S-F using their data from both reflection and transmission measurements.

As an additional check on the theory, for example, to show that the free-carrier dispersion can completely account for the observed reflectivity, a theoretical reflection curve can be generated using the effective mass calculated from Eq. (9). When this is done, one finds that the reflectivity curve, reported by S-F for the *n*-type sample of germanium considered above, is reproduced as exactly as it is possible to determine the values from their report. (The values calculated to achieve this fit are shown in Fig. 1.) Moreover, such a precise fit indicates that we are apparently justified in the substitution of $\langle\tau\rangle$ into Eqs. (2) and (3) and that in this case at least such a substitution produces negligible error. [This result should be compared with Fig. 4a (Appendix A) for $\Omega=3.6$, $\eta=-2$, and $s=-\frac{1}{2}$.]

Before concluding this discussion, it is of interest to consider the effect of various simplifying assumptions upon Eq. (9). For example, the plasma frequency is oftentimes obtained approximately by assuming that $n=1$ [see Eq. (6)], that k is negligibly small compared with unity, and that Ω is very much greater than unity. Under such conditions Eq. (7) can be solved directly to give the result

$$m_c = Ne^2/\omega_0^2\epsilon_0(\epsilon_\infty - 1). \quad (10)$$

This expression is sometimes even further simplified by neglecting unity with respect to ϵ_∞ which yields

$$m_c = Ne^2/\omega_0^2\epsilon_0\epsilon_\infty. \quad (11)$$

Now, if the conductivity effective mass is calculated, for the *n*-type sample of germanium considered here, by means of Eqs. (10) and (11), the resulting values are $m_c^*=0.123$ and 0.115 , respectively. These values differ significantly from the value $m_c^*=0.15$ obtained by Spitzer and Fan. This difference serves as justification for the derivation of the more exact Eq. (9).

Finally, it should be noted that Eq. (9) can be very well approximated by a quadratic equation since the constant term is oftentimes negligibly small.

$$m_c^{*2} \frac{3\epsilon_\infty - 1}{4\epsilon_\infty(\epsilon_\infty - 1)} \frac{(1/\Omega^2) + 5 + 8\Omega^2}{(1 + 3\Omega^2)} C m_c^* + \frac{3\epsilon_\infty - 2}{2\epsilon_\infty(\epsilon_\infty - 1)^2} \frac{(1 + 2\Omega^2)}{(1 + 3\Omega^2)} C^2 = 0. \quad (12)$$

This equation can be solved directly, with the result that there are apparently two solutions. In actual fact, there is only one allowable solution. The root with the plus sign must be chosen, because it is the root which

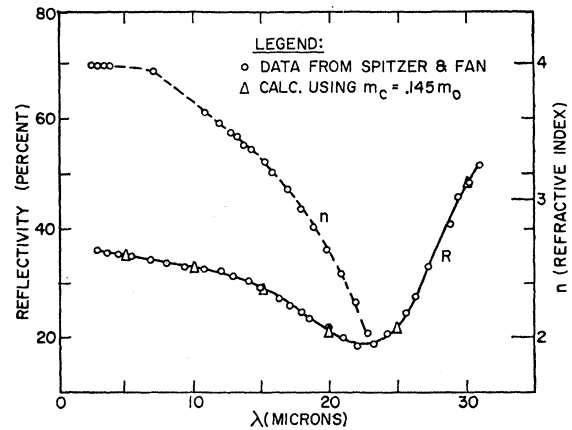


FIG. 1. Reflectivity and refractive index as functions of wavelength for *n*-type germanium having $3.9 \times 10^{18} \text{ cm}^{-3}$ as reported in Ref. 4. In addition, theoretical values for the reflectivity, having been calculated using an effective mass $m_c = 0.145 m_0$, are shown.

in the limit of $\Omega^2 \gg 1$ reduces to the known result expressed in Eq. (10).

COMPARISON WITH OTHER TECHNIQUES

Cyclotron resonance techniques for the determination of the components of the conductivity effective mass have yielded a very great insight into the band structure of germanium and silicon (Ref. 12 gives a good review of this development). However, the condition for the observation of this resonance phenomenon is $\tau\omega_c > 1$, where ω_c is the natural cyclotron frequency of rotation of a moving electron in a magnetic field about an axis parallel to the field, and classically this frequency is given by $\omega_c = eB/m_c$. Consequently, experiments in the microwave region, using reasonable magnetic field intensities, require that τ be 10^{-11} sec or longer. This condition is only fulfilled for high-purity semiconductors at low temperatures.

This limitation severely restricts the usefulness of cyclotron resonance for the determination of m_c . For example, some semiconductors have not as yet been prepared in a sufficiently pure form to enable cyclotron resonance measurements to be made. In this group, for example, is the very interesting family of semiconductors PbS, PbSe, and PbTe. Furthermore, there is reason to believe that the value of the conductivity effective mass in some semiconductors may be temperature-dependent. Evidence of this possibility has been reported in connection with investigations of both germanium¹³ and lead sulfide.¹⁴ However, because of the stringent condi-

¹² T. H. Geballe, *Semiconductors an American Chemical Society Monograph Series*, edited by N. B. Hannay (Reinhold Publishing Corporation, New York, 1959), Chap. 8, pp. 321-6.

¹³ G. MacFarlane, F. McLean, J. Quarrington, and V. Roberts, *Phys. Rev.* **108**, 1377 (1957).

¹⁴ I. Smirnov, B. Moizhes, and E. Nensberg, *Fiz. Tverd. Tela* **2**, 1992 (1960) [English transl.: *Soviet Phys.—Solid State* **2**, 1793 (1961)].

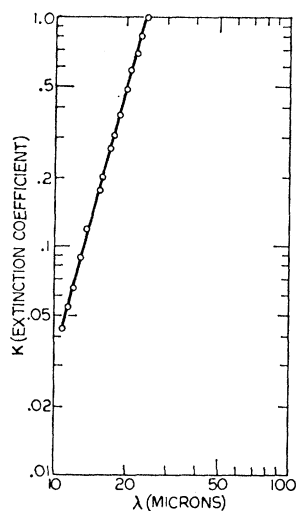


FIG. 2. Absorption index as a function of wavelength for the *n*-type germanium sample of Fig. 1 as reported in Ref. 4.

tion $\tau\omega_c > 1$, cyclotron resonance measurements are generally restricted to liquid-helium temperatures and so they cannot be utilized for the determination of the temperature dependence of m_c .

More recently, techniques have been developed which have utilized measurements in the far infrared. As mentioned earlier (S-F) obtained values of m_c from a combination of reflection and transmission measurements,⁴ and Lax and Wright (L-W) obtained their values from the reflectivity of polarized radiation for several values of magnetic field intensities⁵ (magnetoplasma effect). In both of their analyses (S-F and L-W), the simplification of neglecting the contribution of the lattice to the total dispersion has been inferred.

In connection with the S-F procedure, one notes that if the condition $(\omega\tau)^2 \gg 1$ is fulfilled, the dispersion relation of Eq. (2) is independent of τ . Under this constraint, the conditions placed upon the energy dependence of τ , as noted earlier, do not apply. That is, under the constraint that $(\omega\tau)^2 \gg 1$, the relaxation time may have an arbitrary energy dependence without having any effect on the resultant dispersion relationship, which is

$$n^2 - k^2 - \epsilon_\infty \approx Ne^2/m_c\epsilon_0\omega^2. \quad (13)$$

In addition to the simplified dispersion relation of Eq. (13) and the reflectivity relationship expressed in Eq. (6), the S-F procedure utilizes the fact that the transmission through a sample of thickness x in the absence of interference fringes and for $k^2 \ll n^2$ is given by¹⁵

$$T = (1-R)^2 e^{-\alpha x} / (1-R^2 e^{-2\alpha x}), \quad (14)$$

where α is the linear absorption coefficient which is related to the absorption index k by the relation

$$\alpha = 4\pi k/\lambda. \quad (15)$$

The linear absorption coefficient α equals the reciprocal of the distance within which the incident radiation is attenuated by a factor $(\exp)^{-1}$. The procedure outlined by S-F for the determination of m_c is readily deduced from Eqs. (6), (13)–(15).

This method for obtaining the conductivity effective mass of free carriers is relatively simple compared with cyclotron resonance techniques. It does require that $(\omega\tau)^2 \gg 1$. However, since the frequencies are of the order of 10^{13} – 10^{14} rad/sec, τ may be of the order of 10^{-13} sec compared with 10^{-11} sec for cyclotron resonance. Consequently, measurements can be made on less pure materials and at higher temperatures. Furthermore, it may or may not require transmission measurements. Under certain conditions where k increases very rapidly with wavelength (see Fig. 2), it is possible to consider the reflection region where $k^2 \ll (n-1)^2$ and thus obtain m_c directly from Eqs. (6) and (13). However, one must beware that going to wavelengths much smaller than λ_0 means that $n^2 \rightarrow \epsilon_\infty$ and small errors in the absolute reflectivity can cause very large errors in m^* calculated from Eqs. (6) and (13). Consequently, in general, the S-F procedure requires transmission measurements which in turn require the preparation of very thin samples. For their transmission measurements, S-F used samples whose thickness was of the order of 10μ . Such thicknesses represent a few thousand atomic layers and are very often difficult to achieve without producing extensive damage to the remaining material. For example, it has been possible to achieve such thicknesses in lead telluride only with very careful grinding and polishing techniques and then only very small surface areas ($\approx 600 \times 50 \mu$) were obtained so that special optical techniques must be utilized.¹⁶

The other procedure mentioned above, the measurement of the magnetoplasma reflection as proposed by L-W,⁵ eliminates the necessity of making transmission

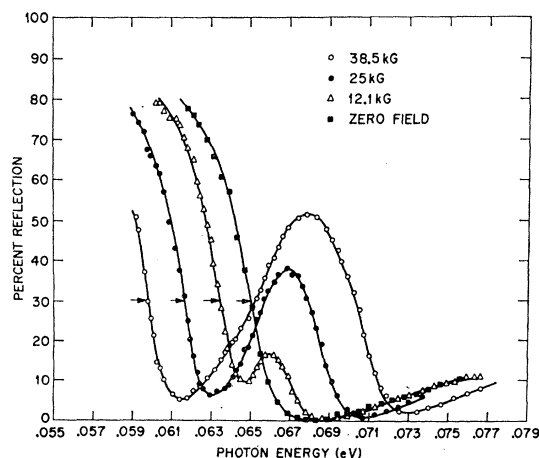


FIG. 3. Magnetoplasma effect in *n*-type InSb having $1.8 \times 10^{18} \text{ cm}^{-3}$ as reported in Ref. 5.

¹⁵ Reference 6, pp. 13–14.

¹⁶ W. W. Scanlon, Phys. Chem. Solids 8, 423 (1959).

measurements. This method utilizes the fact that, in the presence of properly polarized radiation, a magnetic field shifts the minimum in the reflectivity to both sides of the zero-field minimum. This results in the appearance of two minima. The displacement of these minima to either side of the zero-field minimum is directly proportional to the cyclotron resonance frequency $\omega_c = eB/m_e$. In their investigation, L-W have determined the shift as a function of magnetic field strength and have determined m_e from the slope of this dependence. Their data for the magnetoplasma effect in *n*-type indium antimonide are shown in Fig. 3.

This procedure, even though it offers the advantage of eliminating transmission measurements, does contain several inherent drawbacks. For example, magnetic field strengths of 25-kG or less would produce a shift in the minimum of approximately $\pm \frac{1}{2} \mu$ for the germanium sample of Fig. 1. Consequently, for zero-field reflection minima which are shallow and which have a width of a couple of microns, as for example the reflectivity minimum observed by S-F for germanium at room temperature in Fig. 1, the actual shift of the minimum is very difficult to measure. In fact, in their original article Lax and Wright found that the positions of the shifted minima were sufficiently indistinct so that it was expedient to determine the shift from that of the reflection edge on the long-wavelength side of the reflection minimum at an isoreflexion point. This shift is that indicated by the arrows in Fig. 3. However, this latter method for obtaining the shift of the reflection minimum is also subject to certain disadvantages. First of all, it is not clear that the edge should shift the same amount as the minimum since the latter shift was determined assuming that losses are negligible. This is certainly not the case in the region of the reflection edge on the long-wavelength side of the reflection minimum because in this region we have the condition $k > n$. Furthermore, because k increases very rapidly in this region of the reflectivity, the depth of penetration of the incident radiation becomes increasingly shallow. For example, from Eq. (15) we see that the penetration depth ($1/\alpha$) for $\lambda = 30 \mu$ and $k \approx 2$ is only of the order of 1μ . Consequently, the surface preparation becomes increasingly important in this region of the reflectivity curve. Surface damage can therefore distort the reflectivity in this wavelength range. Such surface effects make it even more difficult to determine the relationship of the observed shift in the reflection edge to the cyclotron resonance frequency.

CONCLUSIONS

The relationship between m_e and ω_0 presented in Eq. (9) offers some unique advantages in the determination of m_e . It has rather widespread applicability since it is valid for totally degenerate statistics and for almost all degrees of degeneracy when $|s| \leq \frac{1}{2}$, where s is the power of the energy dependence of τ . Equation (9) is not limited by any assumption that $\omega\tau > 1$. It is much

more applicable than the simplified relation of Eq. (11). It eliminates the necessity of determining the transmission through very thin samples of material on the one hand and the necessity of applying large magnetic fields perpendicular to the direction of propagation of polarized infrared radiation on the other. What is required is the measurement of the infrared reflectivity in order to determine the frequency of minimum reflection. It is not even necessary to obtain absolute reflectivity data. Relative values can serve equally well. That is, a percentage of the incident and/or of the reflected radiation can be lost through experimental error. Although this will result in a decrease in the resolution of the minimum, if the minimum is nevertheless discernible, it is still possible to determine m_e from Eq. (9). Moreover, this procedure is not overly sensitive to the effects of surface damage on the long-wavelength side of the reflectivity minimum. It has been noted above that the surface preparation becomes increasingly important in this region because of the decreasing depth of penetration into the sample. A damaged sample surface has the tendency of dispersing the radiation thereby reducing the reflectivity in this long wavelength region and of presenting a $\langle \tau \rangle$ which is different from that of the bulk material. The reflectivity in the region of the minimum is also subject to distortion from this mechanism only less so than in the region of the absorption edge because of the increased depth of penetration of the incident radiation.

Consequently, m_e can be obtained using Eq. (9) over the large temperature range in which the reflectivity minimum is measurable. The free-carrier concentration of the semiconductor material of interest is limited only by the requirement of single-carrier conduction, by the requirement that the lattice contribution to the dispersion be relatively unimportant, and by the requirement that the reflectivity apparatus be capable of analyzing the reflection near the free-carrier dispersion minimum. However, these limitations on the applicability of this procedure are also common to the other procedures which utilize infrared radiation.

ACKNOWLEDGMENT

I am very greatly indebted to Professor Arthur C. Smith of the Electrical Engineering Department at the Massachusetts Institute of Technology for his advice on the matters presented here.

APPENDIX A

It is convenient to substitute terms expressed as functions of $\langle \tau \rangle$ into Eqs. (2) and (3). Such a substitution is only approximately equivalent to the exact terms. The nature of the approximation involved can be appreciated by examining the following ratios:

$$\frac{\langle \tau^2 / (1 + \omega^2 \tau^2) \rangle}{\langle \tau \rangle^2 / (1 + \omega^2 \langle \tau \rangle^2)} \quad \text{and} \quad \frac{\langle \tau / (1 + \omega^2 \tau^2) \rangle}{\langle \tau \rangle / (1 + \omega^2 \langle \tau \rangle^2)} \quad (16, 17)$$

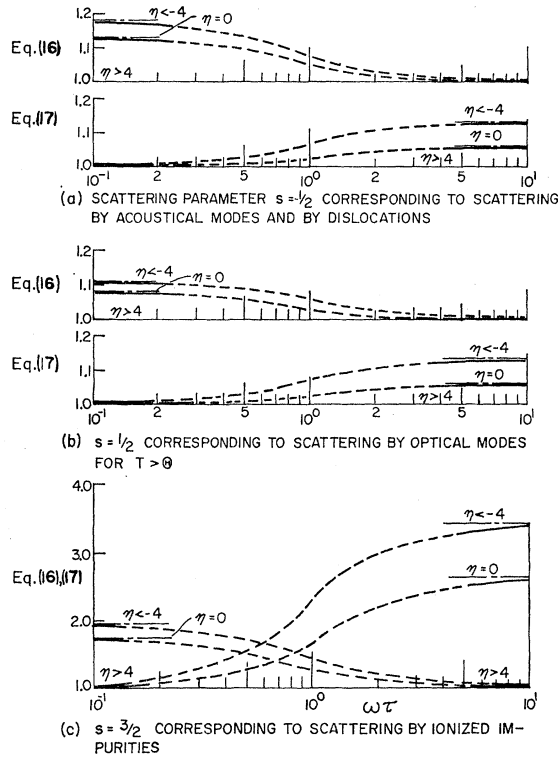


FIG. 4. Asymptotic limits of the ratios (16,17) plotted as functions of $\omega\tau$ for various values of the scattering parameter s and for three degrees of degeneracy $\eta < -4$, $=0$, >4 .

under the conditions of $\omega\tau \gg 1$ and $\omega\tau \ll 1$. Under these conditions the above ratios approach the following limiting values:

$$\begin{aligned} \text{for } \omega\tau \gg 1: & \quad 1 \text{ and } \langle \tau^{-1} \rangle \langle \tau \rangle \\ \text{for } \omega\tau \ll 1: & \quad \langle \tau^2 \rangle \langle \tau \rangle^{-2} \text{ and } 1. \end{aligned}$$

Under these limiting conditions, it is apparent that for complete degeneracy these ratios are unity. In the limit of nondegeneracy these limiting values become

$$\begin{aligned} \text{for } \omega\tau \gg 1: & \quad 1 \text{ and } \Gamma(5/2-s)\Gamma(5/2+s)[\Gamma(5/2)]^{-2} \\ \text{for } \omega\tau \ll 1: & \quad \Gamma(5/2+2s)\Gamma(5/2)[\Gamma(5/2+s)]^{-2} \text{ and } 1 \end{aligned}$$

The asymptotic limits for various scattering mechanisms are shown in Fig. 4 for reduced Fermi potentials of $\eta < -4$, $=0$, and >4 .

Appendix B

The condition $\partial R/\partial\Omega=0$ yields the equation

$$(n^2-1-k^2)n'+2nk'k'=0, \quad (18)$$

where the prime signifies $\partial/\partial\Omega$. Solving for k and k' from Eq. (8) and substituting into (18) yields

$$(n^4-n^2-3\beta^2)n'+2\beta n\beta'=0, \quad (19)$$

where $\beta=B/[2\Omega(1+\Omega^2)]$. Combining Eqs. (7) and (8) yields

$$n^4=\epsilon n^2+\beta^2, \quad (20)$$

where $\epsilon=\epsilon_\infty-[B/(1+\Omega^2)]$, and upon substituting (20) into (19), we obtain

$$[(\epsilon-1)n^2-2\beta^2]n'+2\beta n\beta'=0. \quad (21)$$

By solving (20) for n^2 (choosing the positive root to keep n real), by differentiating this result with respect to Ω , and by substituting $(n^2)'/n$ for n' in Eq. (21), we obtain

$$[(\epsilon-1)n^2-2\beta^2][-2n^2\Omega^3+\beta(1+3\Omega^2)] + 2n^2\beta(1+3\Omega^2)(2n^2-\epsilon)=0. \quad (22)$$

Equation (22) can be rewritten in the form

$$n^4=G(\Omega)n^2+H(\Omega)\beta^2, \quad (23)$$

where $G(\Omega)$ and $H(\Omega)$ are complicated functions of Ω . Equating (23) and (20) yields

$$n^2=\beta^2(H-1)/(\epsilon-G). \quad (24)$$

Dividing Eq. (20) by n^2 and substituting for n^2 from Eq. (24) yields the relationship

$$\beta^2=(\epsilon-G)(H\epsilon-G)/(H-1)^2. \quad (25)$$

After much simplification (25) reduces to Eq. (9) of the text.